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| Calculus 1 |

MAIN BRANCHES OF CALCULUS

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| 1. Limits (approach) |
| 1. Derivatives (change) |
| 1. Integration (anti-derivative) |

AVERAGE RATE OF CHANGE

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| --- |
| ΔY / ΔX |

AROC WITHIN AN INTERVAL

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| --- |
| f(x2) - f(x1) / (x2 - x1) |
| Interval: [x1, x2] |

INSTANTANEOUS RATE OF CHANGE

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| --- |
| = Derivative |
| = Slope of tangent line |

TANGENT LINE

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| --- |
| Slop of a line through 1 point |

SECANT LINE

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| --- |
| Line joining two points |

INSTANTANEOUS RATE OF CHANGE

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| --- |
| [ f(x + h) - f(x) ] / h |

AVERAGE RATE OF CHANGE

|  |
| --- |
| = Secant line |

FIND EQUATION OF TANGENT LINE

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| 1. You need a function and a point 2. Use this equation: [f(x+h) - f(x)] /h 3. Simplify equation & set h to zero 4. Use the point, slope found in step 3, and plug them into point-slope form: (y - yo) = m (x - xo) |

LIMIT LAWS

|  |
| --- |
| lim f(x) = L  x -> c  lim g(x) = M  x -> c |
| 1. **Constant Multiple Rule**   lim [k\*f(x)] = k\*L  x -> c |
| 1. **Sum Rule**   lim [f(x) + g(x)] = L + M  x -> c |
| 1. **Difference Rule**   lim [f(x) - g(x)] = L - M  x -> c |
| 1. **Product Rule**   lim [f(x) \* g(x)] = L \* M  x -> c |
| 1. **Quotient Rule**   lim [f(x) / g(x)] = L / M (M≠0)  x -> c |
| 1. **Power Rule**   lim [f(x)]^n = L^n  x -> c  (n is a positive integer) |
| 1. **Root Rule**   lim n√f(x) = n√L = L^ (1/n)  x -> c  (n is a positive integer) |
| Note: If n is even, we assume that f(x) >= 0 for x in an interval containing c. |

WHEN DO LIMITS NOT EXIST

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| 1. **When a function does not approach the same value from the right and left side**. \* This does not mean that the function is continuous or not. It is just if the function doesn’t approach the same value from both sides. |
| 1. **Certain types of functions**: 2. Step functions 3. A function that grows too large or is not bounded 4. Oscillating functions |

HOW TO FIND FUNCTION’S INVERSE

|  |
| --- |
| 1. Swap x & y 2. Solve for x |

HOW TO FIND FUNCTION’S RECIPROCAL

|  |
| --- |
| 1/ f(x) |

SANDWHICH THEOREM

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| --- |
| @ x = c  For g(x) <= f(x) <= h(x) |
| lim g(x) = lim h(x) = L  x -> c x -> c |
| Then find . . .  lim f(x) = L  x -> c |

FINDING A LIMIT WHEN X -> ?

|  |
| --- |
| 1. See if you can plug the value x is approaching without changing anything |
| 1. The two things that would make the function undefined in step 1 are: 2. Dividing by zero 3. Taking even root of a negative # |
| 1. Use one of the limit tricks to solve for a limit |

LIMIT TRICK

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| 1. Multiply the function by a conjugate   Ex: (a+b) \* (a - b) = a^2 + b^2 |

LIMIT TRICK FOR LONG TERM BEHAVIOR

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| Given:  lim f(x) / g(x)  x -> +/- infinity |
| This only works if x approaches positive or negative infinity |
| 1. Divide f(x) (numerator) by the highest power of x in g(x) (denominator) |
| 1. Divide g(x) (denominator) by the highest power of x in f(x) (numerator) |
| 1. All terms 1/x^r disappear (they become irrelevant) |
| 1. Do some cancelling / algebra to do problem |
| 1. Get the answer |

LONG TERM BEHAVIOR RULES

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| 1. Constants (values without an x) go to zero |
| 1. Terms like 1/x^r or 1/e^infinity |

LONG TERM BEHAVIOR OF RATIONAL FUNCTIONS

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| --- | --- |
| Rational Function =  polynomial / polynomial | |
| Larger power in numerator | Approaching either +/- infinity |
| Larger power in denominator | Approaching zero |
| Equal powers in denominator & numerator | Approaching leading coefficient in numerator divided by leading coefficient in denominator |

BEHAVIOR OF FRACTIONS & LONG TERM BEHAVIOR

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| --- |
| 1/ small input = huge positive output (+ infinity) |
| 1/ huge input = small output |

LOCAL BEHAVIOR

|  |
| --- |
| Area around a point |

LONG TERM BEHAVIOR

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| --- |
| What happens to the y-values as x gets bigger and bigger |
| Ultimate fate of f(x) / end behavior |

UNIQUE LIMIT EXAMPLE

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| calc1 study sheet pic 1 |
| There is no limit for this function because we can’t say that they will join, but they approach the same limit so we can write this:  lim 1/ x^2 = + infinity  x -> 0 |

ONE SIDED LIMIT NOTATION

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| --- | --- |
| lim f(x)  x -> c^+ | Investigate f(x) for values of x approaching c from the right |
| lim f(x)  x -> c^- | Investigate f(x) for values of x approaching c from the left |

AVERAGE SPEED OF FALLING OBJECT

|  |
| --- |
| = distance traveled / elapsed time |
| = [ f(t2) - f(t1)] / [ t2 - t1] |

TYPES OF DISCONTINUITIES

|  |  |
| --- | --- |
| 1. Removable discontinuity | removable discontinuity |
| 1. Jump discontinuity | jump discontinuity |
| 1. Infinite discontinuity (asymptotes) | infinite discontinuity |
| 1. Oscilating discontinuity | oscilating discontinuity |

CONTINUITY DEFINITION

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| lim f(x) = f(a)  x -> a |
| Function is continuous if there are no breaks, stops, or holes |
| 1. Cannot contain any negative exponents 2. No fractional exponents 3. No radicals (square roots, etc.) |

TYPES OF CONTINUOUS FUNCTIONS

|  |  |
| --- | --- |
| **TYPE** | **EXAMPLES** |
| 1. Trigonometric | sin, cos, tan, etc. |
| 1. Polynomial | x^2 + x + 1 |
| 1. Exponential | e^2x, 5e^x, etc. |
| 1. Logarithmic | log10(x), Ln(x) |

INTERMEDIATE VALUE THEOREM

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| Used when we can’t solve for a limit algebraically (you keep choosing values closer and closer to the point) |
| 1. Establish that f(x) is continuous (for ex: you can state that it is one of the types of continuous functions listed right above this box) |
| 1. Show that f(x) changes sign (positive & negative) on some interval |
| 1. State “By intermediate value theorem” the solution is somewhere on that interval |

DERIVATIVE

|  |
| --- |
| Slope of a curve at point (xo ,f(xo)) |
| Slope of tangent line = derivative |
| Derivative’s power is always 1 power less than the main function |

INSTANTANEOUS RATE OF CHANGE

|  |
| --- |
| = Slope |

HOW TO GET DERIVATIVE

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| --- |
| 1. Get the slope |
| Slope = lim [f(xo +h) - f(xo)] / h as h approaches zero |
| 1. Enter the slope & the point into the point slope form |
| Point-slope form: (y - yo) = m (x - xo) |

FUNCTION & ITS DERIVATIVES

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| --- | --- |
| **EQUATION** | **WHAT IT TELLS US** |
| Function | Location / points |
| 1st Derivative | Slope / Change |
| 2nd Derivative | Bendiness / Concavity |

DERIVATIVE NOTATIONS

|  |  |  |
| --- | --- | --- |
| dy/  dx | df/  dx | (d/dx)f(x) |
| derivative notation | The line in the pic to the left means to evaluate at a particular value. | |
| f’(x) = derivative of f(x) | | |

DERIVATIVE RULES

|  |  |
| --- | --- |
| 1. Power rule | x^n = n\*x^(n-1) |
| 1. Constant multiple rule | C\*f(x) = c\*f’(x) |
| 1. Derivative sum rule (sum & difference) | (u+/-v) = du +/- dv |
| 1. Constant rule | If f(x) = c  then f’(x) = 0 |
| 1. Derivatives of exponential functions | For e functions:  Derivative of e^x = e^x  Note: If x is positive |
| All other exponential functions | If f(x) = a^x  f’(x) = (a^x)\*(ln(a)) |
| 1. Product rule | If y = f(x)\*g(x)  y’ = f’(x)\*g(x)+f(x)\*g’(x) |
| 1. Quotient rule | y = f(x) / g(x)  y’ =  g(x)\*f’(x)-g’(x)\*f(x) / [g(x)]^2  Lo\*DHi - Hi\*DLo/  Lo^2 |

NORMAL LINE

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| 1. A normal line is perpendicular line to the tangent line 2. Perpendicular line has slope of negative reciprocal |

UNIQUE INSTANCES OF DERIVATIVES WITH E

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| --- | --- | --- |
| e^-x | = | 1/ e^x |
| Finding the derivative of this would require the quotient rule | | |
| e^2x | = | (e^x)\*(e^x) |
| In this example, we would need the product rule to find the derivative of this function. | | |

DERIVATIVE OF E WITH NEGATIVE EXPONENT

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| --- |
| y’^(B) = [A^B]\*[e^(Ax)] |
| 1. You multiply e^(Ax) by A to the exponent equal to the # of times you are taking a derivative. 2. Remember the negative exponent from e (e^-x) will be brought down with A each time. |
| EX: Derivative of e^-x = -e^-x |

DERIVATIVE OF E WITH STUFF IN EXPONENT

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| --- |
| f(x) = e^2x |
| f’(x) = 2e^2x |
| f’’(x) = 4e^2x |

REMEMBER

|  |
| --- |
| pi is a constant |

DERIVATIVE AS A RATE OF CHANGE

|  |  |  |
| --- | --- | --- |
| Function | s(t) | Position |
| 1st derivative | v(t) | Velocity |
| 2nd derivative | a(t) | Acceleration |
| 3rd derivative | j(t) | Jerk |

DERIVATIVE ELEMENTS

|  |  |  |
| --- | --- | --- |
| v(t) | Δ position/ time | ds/dt |
| a(t) | Δ velocity/ time | d2s/dt2 |
| j(t) | Δacceleration/time | d3s/dt3 |

DERIVATIVE MEASUREMENTS

|  |  |
| --- | --- |
| Velocity | meter/second |
| Acceleration | meter/second^2 |
| Jerk | meters/second^3 |

CHANGING VELOCITY

|  |  |
| --- | --- |
| Increasing velocity | Speeding up |
| Decreasing velocity | Slowing down |
| Zero velocity | No movement |
| Negative velocity | Going backwards |

INSTANTANEOUS VS OVER TIME

|  |  |
| --- | --- |
| Instantaneous | dv/dr |
| Approximation over time | Δ v / Δ r |
| Dv/dr = volume change with respect to the radius | |

AVERAGE VELOCITY

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| --- |
| [V(start time) + V(end time)] / 2 |

SPEED

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| --- |
| Absolute value of velocity. (speedometer) |

ACCELERATION

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| --- |
| Shows the force moving on the object in question. |

CHANGING ACCELERATION

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| --- | --- |
| Negative acceleration | Slowing down |
| Positive acceleration | Speeding up |
| Zero acceleration | Keeping same speed |

BODY’S DISPLACEMENT

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| Distance of end point from starting point. |

AREA OF A CIRCLE

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| Area = [(pi)/4]\*[Diameter^2] |

RATE OF CHANGE OF AREA WITH RESPECT TO CIRCLE’S DIAMETER

|  |  |  |
| --- | --- | --- |
| [dA/dD]= | [(pi)/4]\*2D = | [(pi)\*D]/2 |

CHANGE IN VOLUME

|  |
| --- |
| Derivative \* Δ r |

4 CYCLE OF SIN & COS

|  |
| --- |
| f(x) = sin(x) |
| f’(x) = cos(x) |
| f’’(x) = - sin(x) |
| f’’’(x) = - cos(x) |
| f’’’’(x) = -(-sin(x)) =sin(x) (back 2 top) |

DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

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| --- |
| 1. Derivative of sine(x) = cos(x) |
| 1. Derivative of cos(x) = - sin(x) |
| 1. Derivative of tan(x) = sec^2(x) |
| 1. Derivative of cot(x) = -csc^2(x) |
| 1. Derivative of sec(x) = sec(x)tan(x) |
| 1. Derivative of csc(x) = -csc(x)cot(x) |

WHEN WE CAN SEPARATE TERMS TO EQUAL ZERO

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| EX: (x-1)(x+3) = 0 |
| We can only set each item equal to zero when the product of 2 things equals zero without any extra constants/ values without x |

CHAIN RULE WITH MULTIPLE LAYERS

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| y’ = f’(g(h(x)))\*g’(h(x))\*h’(x) |

IMPLICIT DIFFERENTIATION

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| Get the derivative of an equation that is not a function. |
| 1. Re-write every y as f(x) |
| 1. Take derivative of every term with respect to x. |
| 1. Every time you differentiate a term with f(x), use the chain rule. This will generate a few f’(x) or y’ terms. |
| 1. Solve the resulting equation for f’(x) [or y’ or dy/dx or whatever symbols you are using] |

DERIVATIVES OF COMPOSITION FUNCTIONS

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| --- |
| If y = f(g(x):  Derivative of y = f’(g(x)\*g’(x)  Aka: Derivative of f(x) with normal g(x) times the derivative of g(x) |

DERIVATIVES OF LOGARITHMS

|  |  |
| --- | --- |
| y = ln(x) | y’ = 1/x |
| y = ln(f(x)) | y’ = [1/f(x)]\*f’(x) |
| Y = loga(x)  a = base  base e = ln | y’ = (1/x)\*(1/ln(a)) |
| y = loga(f(x)) | Y’ = (1/f(x))\*f’(x)\*(1/ln(a)) |

REMEMBER LOGS ARE INVERSES OF EXPONENTIAL FUNCTIONS

|  |  |  |  |
| --- | --- | --- | --- |
| When we use implicit differentiation to find y’ | | | |
| Original | | y = e^x | |
| Inverse | | x = e^y | |
| y’=(1/e^y) | e^y = x | | y’ = (1/x) |
| x = e^y | --------> | | 1=e^y\*y |

LOGARITHMIC DIFFERENTIATION

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| --- |
| 1. Using Ln on both sides |
| 1. Use log rules to make the problem more simple |
| 1. Take the derivative of each part |

FOR LOG DIFFERENTIATION

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| --- |
| Ln(y) = (1/y)\*y’ |

RELATED RATES EXPLAINED

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| An equation that relates how our variables change with respect to a new variable time. |

LINEARIZATION

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| Given any function f(x), the linearization around x = a is:  L(x) = f’(a)(x-a) + f(a) |
| Note: (y - yo) = m (x - xo) is same as  y = m (x - xo) + yo |
| Values close to x = a functions is approximately equal to the tangent line |

WHAT IS NEEDED FOR LINEARIZATION

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| 1. A point: (a, f(a)) |
| 1. A slope: m = f’(a) |

LINEARIZATION OF (1+x)^K

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| --- |
| Approx. = (1+kx) when x near zero |

ACCEPTABLE ERROR

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| The acceptable range for linearization |
| Absolute value of (y-value for original equation) - (x-value) |

PERCENT ERROR

|  |
| --- |
| [(exact - approximate)/(exact)]\*100 |

TYPES OF DERIVATIVE Y=0 SPOTS

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| 1. **Resting points**: Both sides (left & right) of a point are increasing/ both decreasing |
| 1. **Does not exist**: A hole in the spot, other non-continuous behaviors |
| 1. **Max or min**: When left and right are going opposite directions and the spot does exist. |
| Global max & mins and local max & mins are different |
| Asymptotes only occur with open intervals |
| Not all functions have absolute max or mins or local max or mins |
| Exponential functions are never equal to zero |

CRITICAL POINTS

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| --- |
| f’(x) = 0 |
| Not all critical points are extreme places |

VOCAB

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| 1. **Values**: Function outputs or y-values |
| 1. **Functions**: Specific relationship between x & y |
| 1. **Closed Interval**: An interval that includes end points |

HOW TO FIND INFLECTION POINTS

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| --- |
| 1. Find 1st derivative |
| 1. Find 2nd derivative |
| 1. Set 2nd derivative to zero |
| 1. See if signs change on each side |

FIND ABSOLUTE MAX/MIN OF FUNCTION

|  |
| --- |
| 1. Find derivative |
| 1. Solve for f’(x) = 0 (y=0 for derivative function) |
| 1. Make a list of x-values for f’(x) = 0 |
| 1. Check if f’(x) DNE in some places |
| 1. Enter values around points from step 3 in f’(x) to see if the output is positive/negative to check if left & right points are going in opposite directions |
| 1. Enter step 3 values into original function to find points |
| 1. Smallest output = absolute min & largest output = absolute max |

FIND ALL REGIONS OF CONCAVITY

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| --- |
| 1. Find 1st & then 2nd derivative |
| 1. Set 2nd derivative to zero |
| 1. Check if 2nd derivative positive & negative signs on each side of points where f’’(x) = zero |
| 1. f’’(x) positive = f(x) concave down   f’’(x) negative = f(x) concave up |

FIND ALL REGIONS WHERE FUNCTION IS INCREASING & DECREASING

|  |
| --- |
| 1. Find 1st derivative f’(x) |
| 1. Equal 1st derivative to zero |
| 1. Find values in regions around points found on step 2 to see if they are positive or negative 2. f’(x) negative = f(x) decreasing   f’(x) positive = f(x) increasing |

RELATIONSHIP BETWEEN FUNCTIONS & DERIVATIVES

|  |  |
| --- | --- |
| Negative derivative | Decreasing function |
| Positive derivative | Increasing function |
| Negative 2nd derivative | 1st derivative concave down |
| Positive 2nd derivative | 1st derivative concave up |

DOES NOT EXIST INSTANCES

|  |
| --- |
| 1. Logs of zero |
| 1. Logs of negative numbers |
| 1. Dividing by zero |
| 1. An even square root of a negative # |
| 1. Some periodic equations like the tangent function |

2ND DERIVATIVE

|  |
| --- |
| Tells us how f(x) increases or decreases |

SUCCESIVE APPROXIMATION OF AREA UNDER CURVE OF VELOCITY FUNCTION

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| --- |
| x-values = time  y-values = velocity |
| Height = f(x)  Width = Δx (change in x) |
| Area under curve = velocity\*time = r\*t = distance |
| Area = Height \* Width |

EXACT AREA OF F(X) UNDER INTERVAL [a, b]

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| --- |
| integral notation 3 with equivalent  This example uses dx which means it is with respect to f(x) |
| 1. Find an antiderivative of f(x) & call it F(x) |
| 1. Evaluate the antiderivative at end point & subtract antiderivative at start point: F(b) - F(a) |
| integral notation 2 |
| \* End points a & b are always included  \* [a, b] is interval over which we want an area |
| area summation notation |

INTEGRATION

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| An integral is an anti-derivative |
| With integration we are finding the area between the function (within a given range) and the x -axis. If some of the function dips below the x-axis, then some of the values below the x-axis will negate some of the x-values above the x-axis since the x-values below the x-axis would result in a negative y-output. The exception would be if a problem is asking for the total area, in which you need to take the absolute value of the negative outputs. |
| Integration finds the net area between the function and the x-axis |
| We can only find out if a function goes below the x-axis (if there’s some negative outputs) by graphing the function. |

APPLIED OPTIMIZATION

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| --- |
| 1. Read problem |
| 1. Introduce variables |
| 1. Write equation |
| 1. Test critical points |
| \* Make something biggest/smallest subject to constraints  \* Constraints can be money, etc. |

GENERAL OPTIMIZATION PROBLEMS

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| --- |
| Not having set variables or constraints yet |

L’HOPITAL’S RULE

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| If limit of (f(x)/g(x) as x approaches a results in 0/0 or infinity/infinity, then limit of f(x)/g(x) is same result as the limit of f’(x)/g’(x) as x approaches a. |
| L’Hopitals rule can help with speed, but also there are cases when the algebra techniques wouldn’t work and the rule is needed. |

L’HOPITAL’S RULE STEPS

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| 1. Solve for function |
| 1. If answer equals 0/0 or infinity/infinity, then we can solve for the limit of f’(x)/g’(x) as x approaches a |

INDETERMINATE FORM

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| --- |
| This is when we don’t know what the value is. |
| This is cases like 0/0 or infinity/infinity |

FINDING LIMIT STEPS

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| --- |
| 1. See if x value just plugs in. |
| 1. See if you can use LH rule. |
| 1. See if older ch.2 algebra techniques work. |

ANTI-DERIVATIVES

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| A function is an anti-derivative of the original function if the derivative of the anti-derivative is the original function |
| F(x) = notation for anti-derivative |
| F(x) is anti-derivative if F’(x) = f(x) |
| Anti-derivative = integral |

GENERAL ANTI-DERIVATIVE FORM

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| --- |
| F(x) = ….. + c |

INTEGRAL RULES

(ANTI-DERIVATIVE RULES)

|  |  |
| --- | --- |
| **FUNCTION** | **GENERAL FORM** |
| x^n | [1/(n+z)]\*x^(n+1) + c  n ≠ -1 |
| sin(kx) | -(1/k)cos(kx) + c |
| cos(kx) | (1/k)sin(kx) + c |
| sec^2(kx) | (1/k)tan(kx) + c |
| csc^2(kx) | -(1/k)cot(kx) + c |
| sec(kx)tan(kx) | (1/k)sec(kx) + c |
| csc(kx)cot(kx) | -(1/k)csc(kx) + c |
| e^kx | (1/k)e^(kx) + c |
| 1/x | Ln|x| + c  x ≠ 0 |
| 1/√1-(k^2)\*(x^2) | (1/k)sin^-1(kx) + c |
| 1/(1+(k^2)\*(x^2) | (1/k)tan^-1(kx) + c |
| 1/x√(k^2)\*(x^2)-1 | Sec^-1(kx) + c  kx > 1 |
| a^(kx) | (1/(k\*Ln(a))\*a^(kx) + c  a > 0, a ≠ 1 |

INTEGRAL POWER RULE EXCEPTION

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| --- |
| (1/x) -> Ln|x|  \*(1/x) = x^-1 but doesn’t follow the x^n rule. |

INTEGRAL POWER RULE

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| --- |
| 1. Add 1 to the power |
| 1. Divide the number multiplied by x by the new power (value + 1) |
| 1. Add c to the end |
| 1. Solve for c if you have an (x,y) |
| 5.Take anti-derivative’s derivative & see if it’s the original function. |

PERIODIC FUNCTION INTEGRALS

|  |
| --- |
| 1. Take derivative of periodic function’s outside, but leave inside the same |
| 1. Divide by the same derivative of the inside |
| 1. Add a c (aka + c) to the end. |
| 1. Solve for c if you are given a point |
| 1. Check if you can take derivative of integral and that it returns to the original function. |

DEFINITE VS INDEFINITE INTEGRATION

|  |  |
| --- | --- |
| Definite Integration | When info like start & end point given so we can find the + c |
| Indefinite Integration | When we add the + c to the end |

DEFINITE INTEGRATION

|  |
| --- |
| F(b) - F(a) |
| Integral with end point as x -value minus integral with start point as x-value. |

FUNDAMENTAL THEOREM OF CALC

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| --- |
| This explains how to get an exact area ( accumulation machine) |
| Net change in f(x) on interval [a,b] |
| F(b) - F(a) |

SUM UP ALL INTEGERS BETWEEN A START & END POINT

|  |
| --- |
| area approximation general form |
| [(end point + start point) \* (total integers you’re counting)] / 2 |

WHEN FUNCTIONS & DERIVATIVES CHANGE SIGNS

|  |  |
| --- | --- |
| f(0) | y-intercept |
| f(x) = 0 | x-intercept |
| f’(x) = 0 | f(x) has max or min/ vertex |
| f’’(x) = 0 | f(x) has inflection point |

|  |  |
| --- | --- |
| f(x) positive  f(x) negative | Positive y-value  Negative y-value |
| f’(x) positive  f’(x) negative | f(x) y-values increasing  f(x) y-values decreasing |
| f’’(x) positive  f’’(x) negative | f(x) concave up  f(x) concave down |

|  |  |
| --- | --- |
| f(x) goes from positive to negative & vice versa | f(x) crosses x-axis |
| f’(x) goes from + to - & vice versa | f(x) has max or min |
| f’’(x) goes from + to - & vice versa | f(x) has inflection point |

IMPORTANT IDENTITIES

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| Pythagorean Identity  sin^2(x) + cos^2(x) = 1 |
| Power Reductions  sin^2(x) = [1 - cos(2x)] / 2  cos^2(x) = [1 + cos(2x)] / 2  Sin(2x) = 2 - sin(x)cos(x) |
| sin(2x) / 2(sin(x)) = cos(x) |

1. SUBSTITUTION BASIC FORM

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| u-substitution basic form |

1. SUBSTITUTION STEPS

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| Starting form:  integral of: function \* dx   1. State that you will be using u-substitution |
| 1. Find a portion of the function that is “inside” something or a complex part of the function that has an x in it.Make that equal to u. |
| 1. Get derivative of u. |
| 1. Equal u’s derivative to du/d(variable in function) (du/dx or du/dy). 2. Solve for dx, dy, etc. |
| 1. Put equation from step 5 in place of dx in equation (multiplied by original function with u removed) |
| 1. Cancel out anything that can |
| 1. Simplify |
| 1. If the new function is simple enough, put the thing that “u” took the place of back into the function and take it’s integral. Otherwise, do the steps again. |